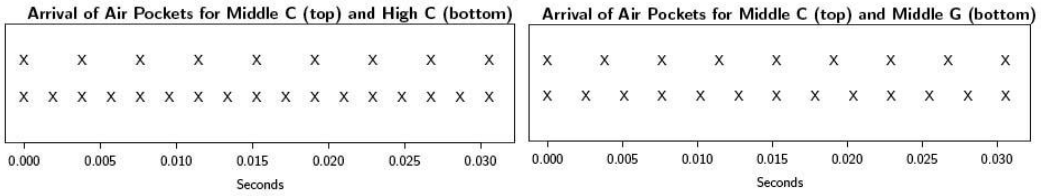
One-Pager “Music and Mathematics”

We would like to tell you about music and mathematics, the most beautiful after love, the most difficult after faithfulness (1)

When two tones or voices, which are one octave apart, sound together, then the Greek call this harmonia. Later Pythagoras changed the word harmonia to octave.

A sound wave creates minute pockets of higher and lower air pressure, and all the sounds we hear are caused by these pressure changes. With music, the frequency at which these pockets strike your ear controls the pitch that you hear.



(left picture) Since the frequency of High C is exactly twice that of Middle C, the two notes line up perfectly. Every two air pocket arrivals for High C correspond perfectly to one arrival for Middle C. (right picture) We see that every second arrival for Middle C, lines up almost perfectly with every third arrival for Middle G. Once again, the two wave patterns fit well together. (2)

Hyppasos asked the question: When I have two tones or two guitar strings, which are the same and which are one octave apart, then they sound good together. Which tone lies in the middle? He answers this question three times; the result is astonishing and wonderful, resulting in our 12 tones system.(3)

The first mean is called the arithmetic mean (AM), we calculate it: From the whole string we take a part away, to the half string we add the same part. The result is 1/3 The whole string is 66 cm long, minus 1/3 (22) gives 44 cm. The half string is 33 cm long, add 1/3 (11) gives 44 so 44 cm is my middle note. Ratio middle to half is 4:3, these sound good together. (note a)The second mean is called the harmonic mean (HM), and we get it by just dividing half the string in two, between the whole string (4/4) and the half string(2/4) lies ¾ of the string, half of 33 is 16.5 cm times 3 is 49,5 cm. Ratio middle to half is 3:2, sounds good together (note b)The third middle is called geometric mean (GM) and is built as follows: take the length of half a string, multiply (enlarge) this length with some factor, to get my new mean (GM). Multiply my new middle length (GM) another time with the same factor to get the length of my whole string again. The outcome is $\sqrt{2}$ (1.41…) (46.53 cm) (note a-sharp)But $\sqrt{2}$ cannot be written down as a ratio of whole numbers. We can only approximate this number, about 1.41. When playing this note, we can hear that, we play around a note, it sounds a little bit “strange”. This number we call irrational. (4) Play the whole string, then the GM, then the half string: these sound strange together, thrilling, intriguing, challenging, provoking, exciting, and moving. Music moves like life and love.

And always when we make music, we play in our head around such strange notes, we fantasize around this irrational number, we try to find whole numbers and ratios, even if they are not there, and that is the reason music is so moving and beautiful. This was the first time in human history that people were thinking about numbers as such, instead of measuring and counting, and from here our Western mathematics evolved, music evocates Greek mathematics. (5)

1 Kittler Musik und Mathematik Hellas 1 Aphrodite 0.1 “Wir möchten euch Musik und Mathematik erzählen: das Schönste nach der Liebe, das Schwerste nach der Treue.
2 https://plus.maths.org/content/magical-mathematics-music
3 http://thinkzone.wlonk.com/Music/12Tone.htm
4 Hyppasos of Metapont, pupil of Pythagoras, 500 before Christ, is musician, mathematician and philosopher. Kittler Friedrich Musik und Mathematik I Hellas 1: Aphrodite 252 Wilhelm Fink 2009
5 Ibid 257